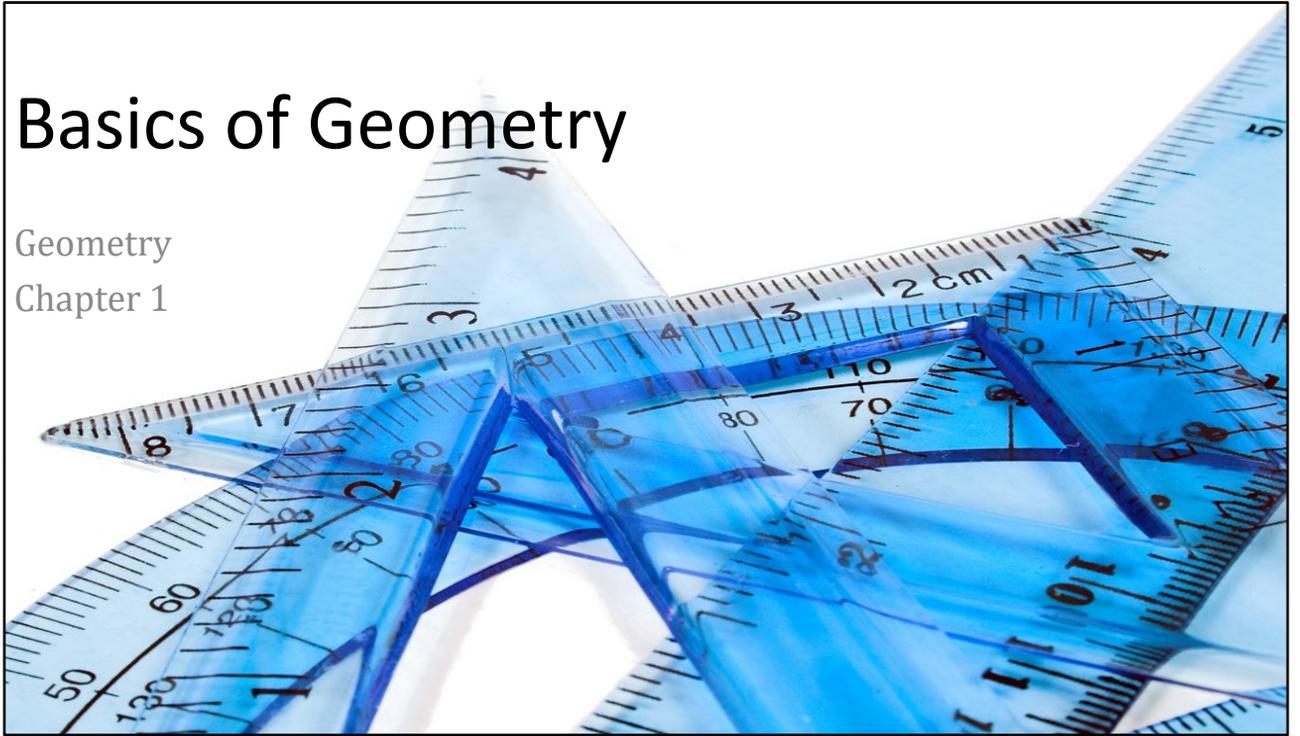


Basics of Geometry

Geometry
Chapter 1



Geometry 1

- This Slideshow was developed to accompany the textbook
 - *Big Ideas Geometry*
 - *By Larson and Boswell*
 - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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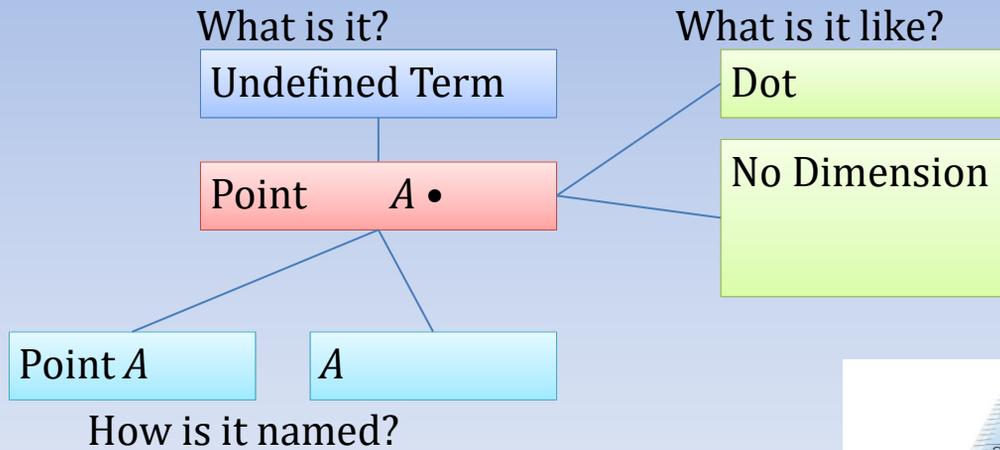
Objectives: By the end of the lesson,

- I can describe a point, a line, and a plane.
- I can define and name segments and rays.
- I can sketch intersections of lines and planes.

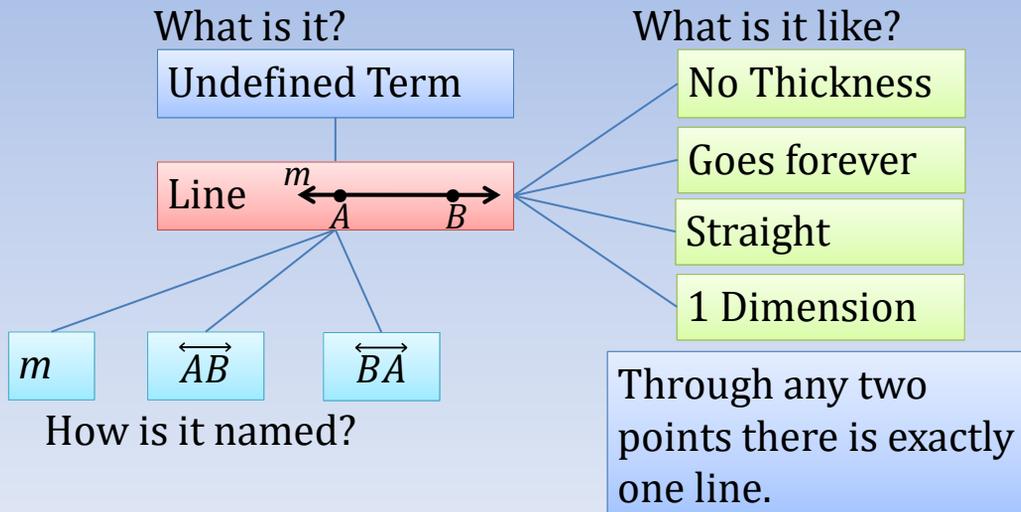
1.1 POINTS, LINES, AND PLANES



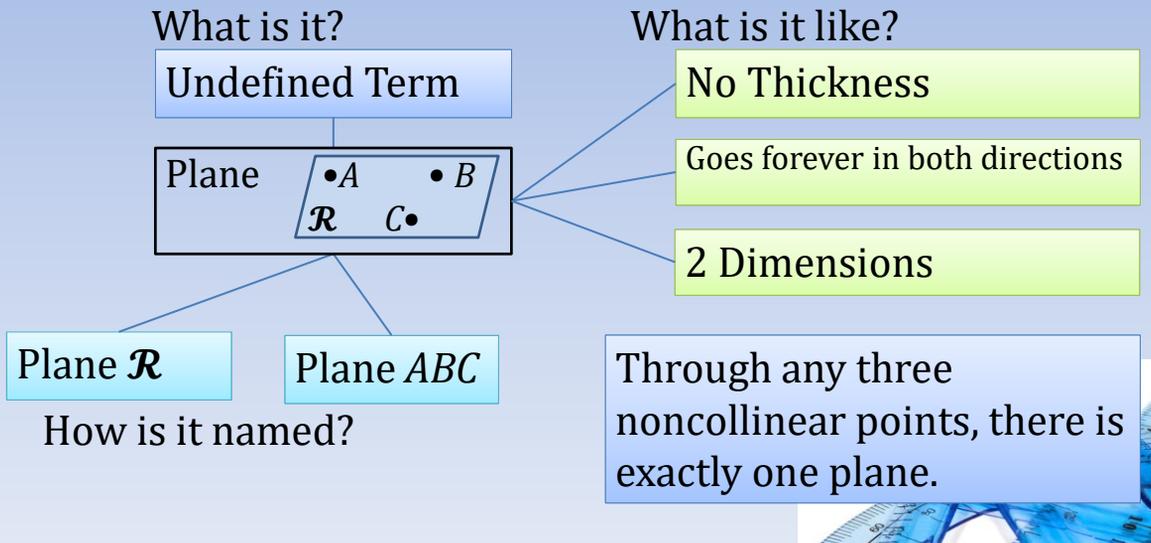
1.1 Points, Lines, and Planes



1.1 Points, Lines, and Planes



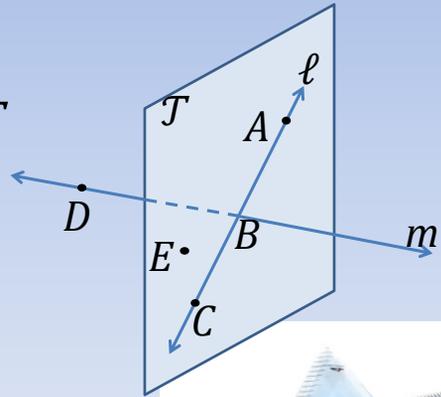
1.1 Points, Lines, and Planes



Book just uses bold capital letter to name a plane.

1.1 Points, Lines, and Planes

- Give two other names for \overleftrightarrow{BD}
- Give another name for plane \mathcal{J}
- Name three collinear points
- Name four coplanar points



$\overleftrightarrow{DB}, m$
Plane ABE
 A, B, C
 A, B, C, E

1.1 Points, Lines, and Planes

What is it?

Part of a Line

Segment 

What is it like?

Part of a line between two endpoints

Does NOT go on forever

Named by endpoints

\overline{AB}

\overline{BA}

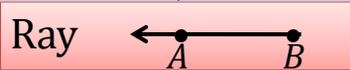
How is it named?



1.1 Points, Lines, and Planes

What is it?

Part of a Line



Named by endpoint followed by one other point

\overrightarrow{BA}

How is it named?

What is it like?

Part of a line starting at one endpoint and continuing forever

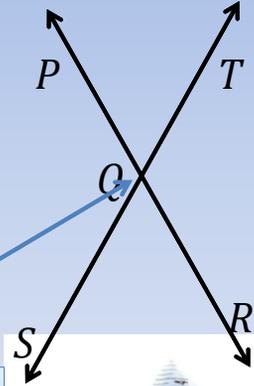
Goes forever in one direction only

If two rays have the same endpoint and go in opposite directions, they are called **opposite rays**.

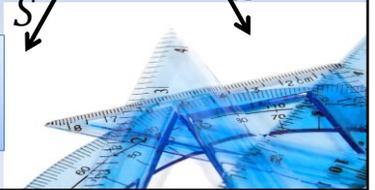


1.1 Points, Lines, and Planes

- Give another name for \overline{PR}
- Name all rays with endpoint Q
- Which of these rays are opposite rays?



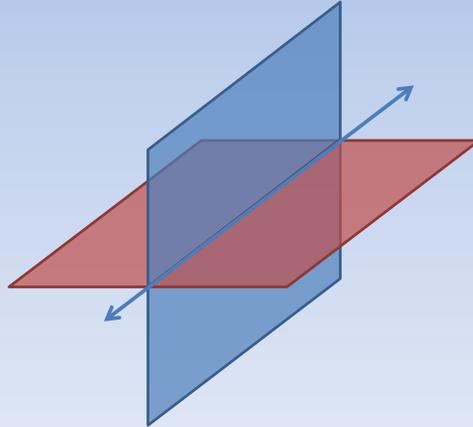
The intersection of two lines is a point.



\overline{RP}
 $\overrightarrow{QP}, \overrightarrow{QR}, \overrightarrow{QT}, \overrightarrow{QS}$
 \overrightarrow{QT} and \overrightarrow{QS} ; \overrightarrow{QP} and \overrightarrow{QR}

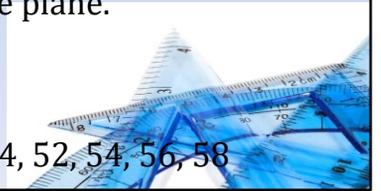
1.1 Points, Lines, and Planes

- The intersection of two planes is a line.



1.1 Points, Lines, and Planes

- Sketch a plane and two intersecting lines that intersect the plane at separate points.
- Sketch a plane and two lines that do not intersect lying in the plane.
- Sketch a plane and two intersecting lines that lie in the plane.
- 8 #2, 4, 6, 8, 10, 12, 14, 18, 20, 22, 24, 26, 28, 30, 32, 34, 52, 54, 56, 58



Objectives: By the end of the lesson,

- I can measure a line segment.
- I can explain and use the Segment Addition Postulate.

1.2 MEASURING AND CONSTRUCTING SEGMENTS

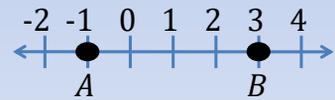


1.2 Measuring and Constructing Segments

- Postulate (or Axiom) – Rule that is accepted without proof
- Theorem – Rule that is proven

Ruler Postulate

Any line can be turned into a number line



1.2 Measuring and Constructing Segments

What is it?

Length measurement

Distance

What is it like?

Difference of the coordinates of the points

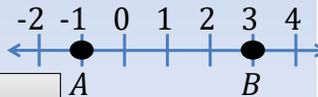
$$AB = |x_2 - x_1|$$

AB

BA

How is it named?

Find AB



$$AB = 3 - (-1) = 4$$

1.2 Measuring and Constructing Segments

What is it?

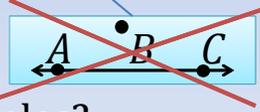
Point Placement

Between

What is it like?

On the segment with the other points as the endpoints

Does not have to be the midpoint



What are examples?

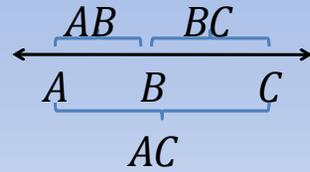


1.2 Measuring and Constructing Segments

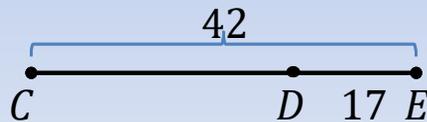
Segment Addition Postulate

If B is between A and C ,
then $AB + BC = AC$

If $AB + BC = AC$, then B is
between A and C



- Find CD



$$CD + DE = CE$$

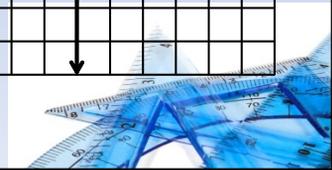
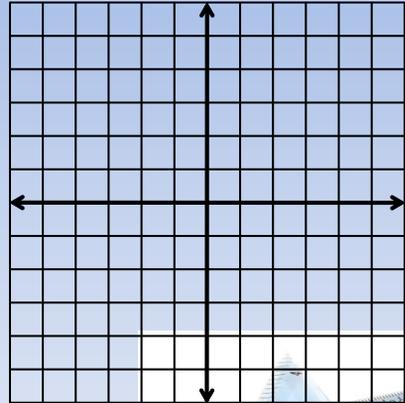
$$CD + 17 = 42$$

$$CD = 25$$

1.2 Measuring and Constructing Segments

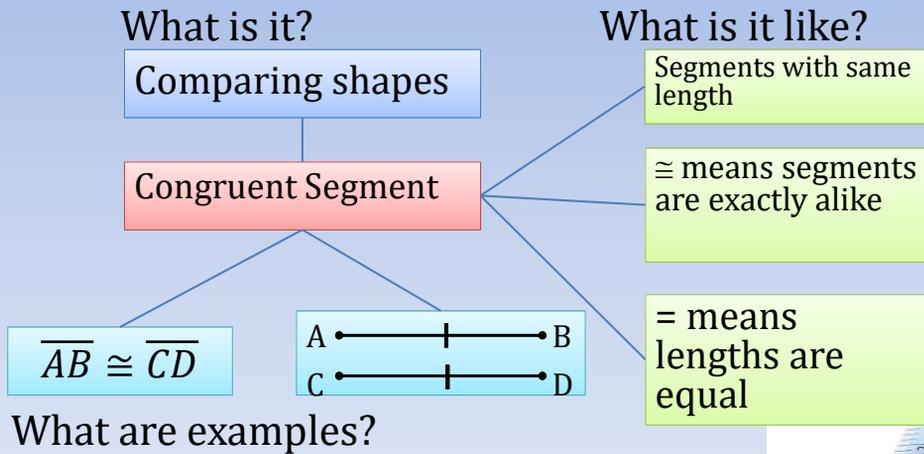
Graph $X(-2, -5)$ and $Y(-2, 3)$.

- Find XY .



$$XY = 3 - (-5) = 8$$

1.2 Measuring and Constructing Segments



What are examples?

16 #2, 4, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 34, 38, 40, 41, 46, 48

Objectives: By the end of the lesson,

- I can find lengths of segments.
- I can find the midpoint of a segment.

1.3 USING MIDPOINT AND DISTANCE FORMULAS



1.3 Using Midpoint and Distance Formulas

What is it?

Part of a Segment

Midpoint



What is it like?

Very middle of the segment

Point that divides the segment into two congruent segments.

M is the midpoint of \overline{AB}

$$\overline{AM} \cong \overline{MB}$$

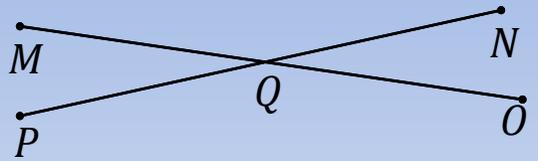
$$AM = MB$$

What are some examples?

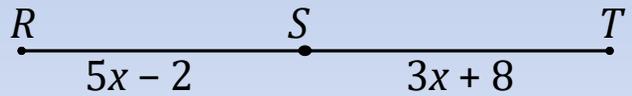
Segment Bisector is something that intersects a segment at its midpoint.

1.3 Using Midpoint and Distance Formulas

- \overline{MO} bisects \overline{NP} at Q . If $PQ = 22.6$, find PN .



- Point S is the midpoint of \overline{RT} . Find ST .



$$PQ = \frac{1}{2}PN$$
$$22.6 = \frac{1}{2}PN$$
$$PN = 45.2$$

$$5x - 2 = 3x + 8$$
$$2x - 2 = 8$$
$$2x = 10$$
$$x = 5$$
$$ST = 3(5) + 8 = 23$$

1.3 Using Midpoint and Distance Formulas

Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- Find the midpoint of $G(7, -2)$ and $H(-5, -6)$



$$\left(\frac{7 + (-5)}{2}, \frac{-2 + (-6)}{2} \right) = (1, -4)$$

1.3 Using Midpoint and Distance Formulas

- The midpoint of \overline{AB} is $M(5, 8)$. One endpoint is $A(2, -3)$. Find the coordinates of endpoint B .



$$(5, 8) = \left(\frac{x + 2}{2}, \frac{y + (-3)}{2} \right)$$

$$\text{x-coords: } 5 = \frac{x+2}{2} \rightarrow 10 = x + 2 \rightarrow x = 8$$

$$\text{y-coords: } 8 = \frac{y-3}{2} \rightarrow 16 = y - 3 \rightarrow y = 19$$

$(8, 19)$

1.3 Using Midpoint and Distance Formulas

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- What is PQ if $P(2, 5)$ and $Q(-4, 8)$?

- 24 #2, 4, 6, 8, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 44, 46, 48, 50, 59



$$PQ = \sqrt{(-4 - 2)^2 + (8 - 5)^2}$$

$$PQ = \sqrt{(-6)^2 + (3)^2}$$

$$PQ = \sqrt{36 + 9}$$

$$PQ = \sqrt{45} = 3\sqrt{5} \approx 6.71$$

Objectives: By the end of the lesson,

- I can classify and describe polygons.
- I can find perimeters of polygons in the coordinate plane.
- I can find areas of polygons in the coordinate plane.

1.4 PERIMETER AND AREA IN THE COORDINATE PLANE



1.4 Perimeter and Area in the Coordinate Plane

What happens when you leave the door of the birdcage open?

What is it?

Shape

Polygon

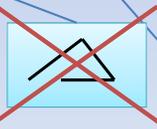
What is it like?

Sides are straight segments

Sides only intersect at ends

Closed shape

No curves



What are examples?

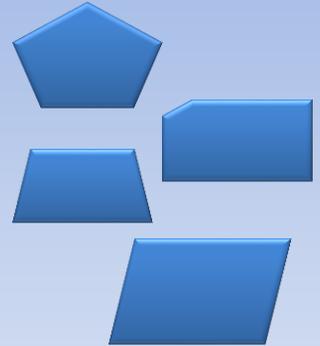


You get a Polly gone

1.4 Perimeter and Area in the Coordinate Plane

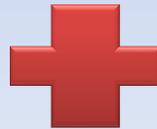
Convex

All angles poke out of shape.
A line containing a side does NOT go through the middle of the shape.



Concave

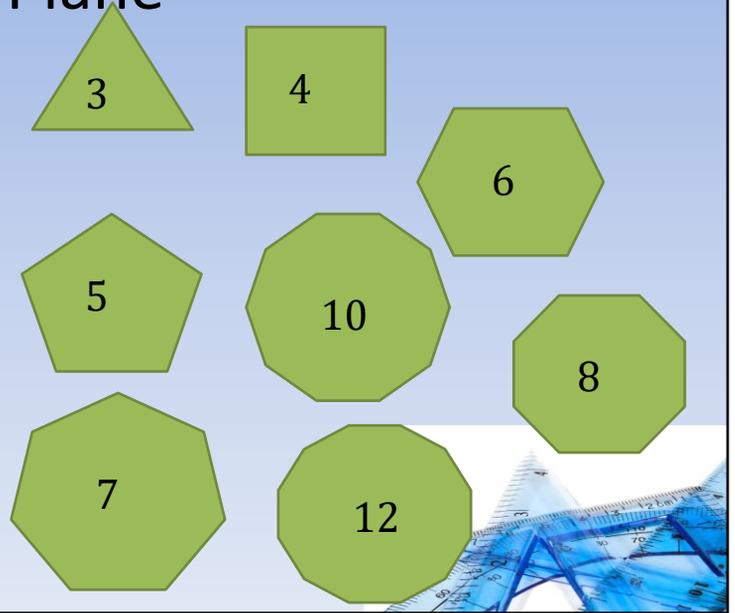
Not convex. (There's a "cave".)



1.4 Perimeter and Area in the Coordinate Plane

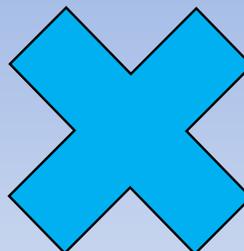
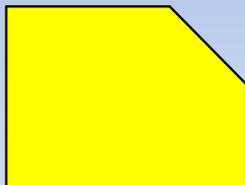
Number of sides	Type of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
13	13-gon
n	n-gon

Plane



1.4 Perimeter and Area in the Coordinate Plane

- Classify each polygon by the number of sides. Tell whether it is *convex* or *concave*.



Pentagon, convex

Dodecagon, concave

1.4 Perimeter and Area in the Coordinate Plane

Perimeter (P)

Distance around a figure

Circumference (C)

Perimeter of a circle

Area (A)

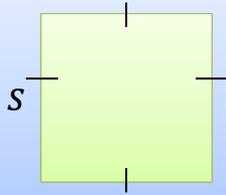
Amount of surface covered by a figure



1.4 Perimeter and Area in the Coordinate Plane

Square

Side s



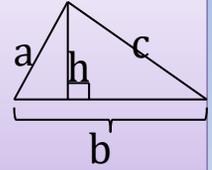
- $P = 4s$

- $A = s^2$

Triangle

sides a, b, c

base b , height h



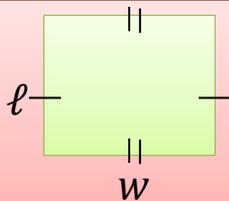
- $P = a + b + c$

- $A = \frac{1}{2}bh$

Rectangle

Length ℓ

Width w



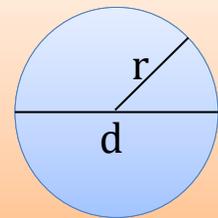
- $P = 2\ell + 2w$

- $A = \ell w$

Circle

diameter d

radius r

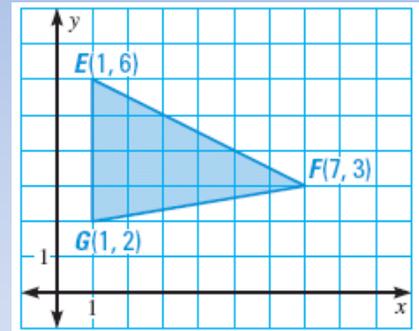


- $C = 2\pi r$

- $A = \pi r^2$

1.4 Perimeter and Area in the Coordinate Plane

- Describe how to find the height from F to \overline{EG} in the triangle.
- Find the perimeter and area of the triangle.



The height is perpendicular to the base, so it hits EG at $(1, 3)$. Distance from $(1, 3)$ to $(7, 3) = 6$

Perimeter: find the lengths of each side

$$EG = 4$$

$$FG = \sqrt{(7 - 1)^2 + (3 - 2)^2} = \sqrt{36 + 1} = \sqrt{37} = 6.08$$

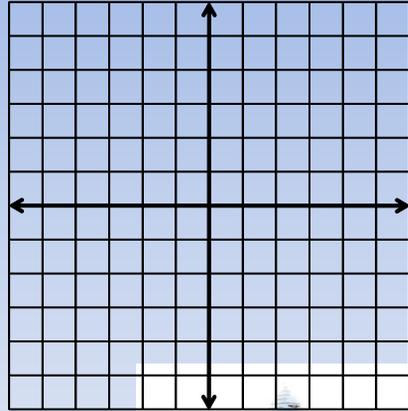
$$EF = \sqrt{(7 - 1)^2 + (3 - 6)^2} = \sqrt{36 + 9} = \sqrt{45} = 6.71$$

$$P = 4 + 6.08 + 6.71 = 16.79$$

$$\text{Area: } \frac{1}{2}(4)(6) = 12$$

1.4 Perimeter and Area in the Coordinate Plane

- Find the area of $\square ABCD$ with vertices $A(1, 3)$, $B(3, -3)$, $C(-2, -3)$, and $D(-4, 3)$.



- 32 #2, 4, 6, 10, 12, 16, 18, 20, 24, 26, 36, 38, 39, 40, 42



$$\begin{aligned}b &= 3 - (-2) = 5 \\h &= 3 - (-3) = 6 \\A &= bh = (5)(6) = 30\end{aligned}$$

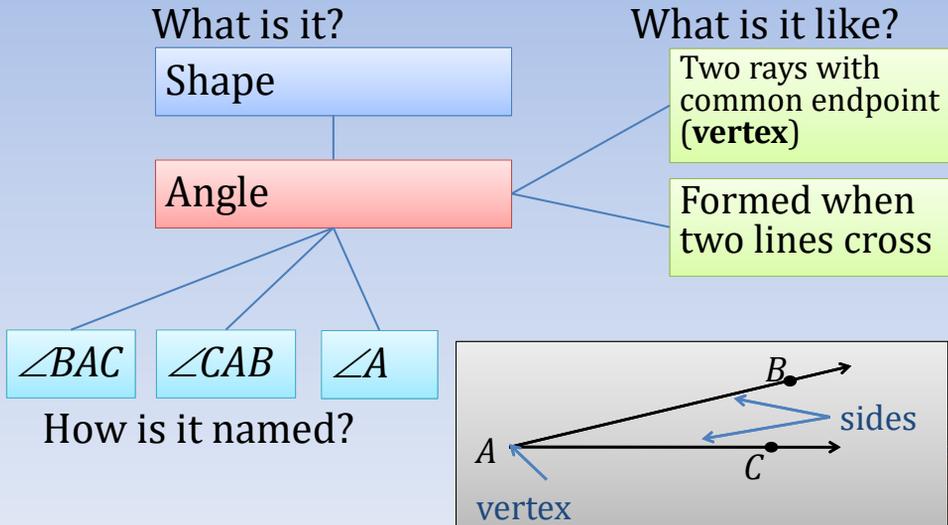
Objectives: By the end of the lesson,

- I can measure and classify angles.
- I can find angle measures.

1.5 MEASURING AND CONSTRUCTING ANGLES



1.5 Measuring and Constructing Angles



1.5 Measuring and Constructing Angles

Protractor Postulate

A protractor can be used to measure angles

What is it?

Angle measurement

Measure

$m\angle A$

$m\angle BAC$

How is it named?

What is it like?

Difference coordinates of each ray on a protractor

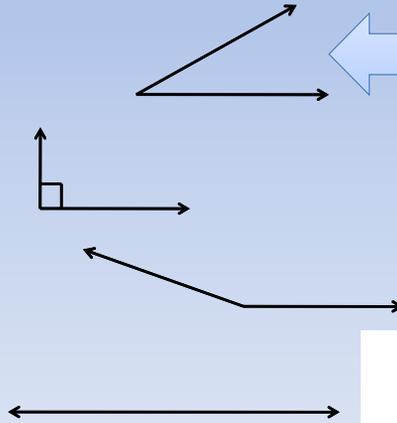
$$m\angle A = |x_2 - x_1|$$



1.5 Measuring and Constructing Angles

- Classifying Angles

- Acute
 - Less than 90°
- Right
 - 90°
- Obtuse
 - More than 90°
- Straight
 - 180°



It's such a cute angle!



1.5 Measuring and Constructing Angles

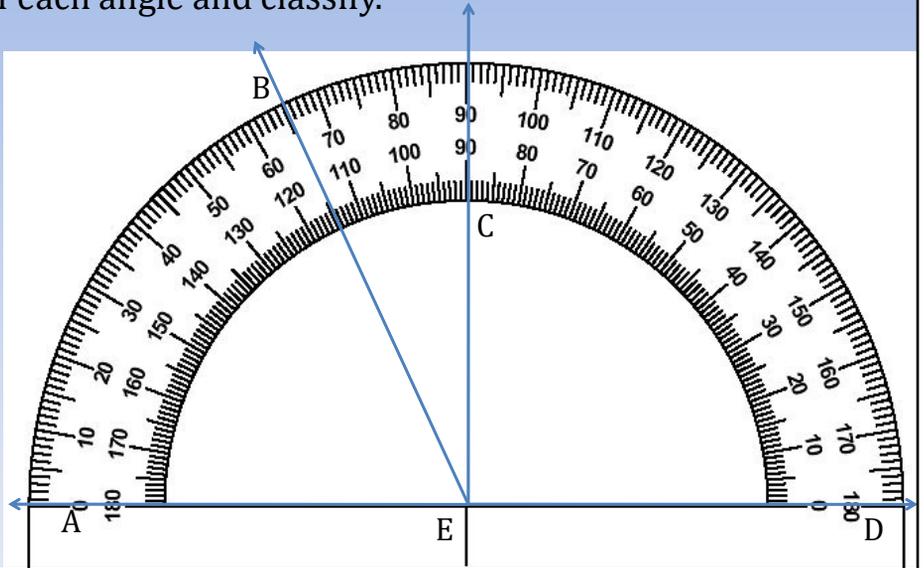
- Find the measure of each angle and classify.

➤ $\angle DEC$

➤ $\angle DEA$

➤ $\angle CEB$

➤ $\angle DEB$



$\angle DEC = 90^\circ$ right

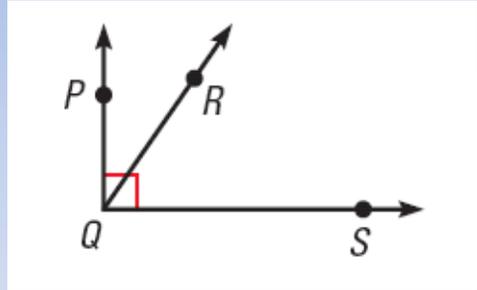
$\angle DEA = 180^\circ$ straight

$\angle CEB = 25^\circ$ acute

$\angle DEB = 115^\circ$ obtuse

1.5 Measuring and Constructing Angles

- Name all the angles in the diagram.
- Which angle is a right angle?



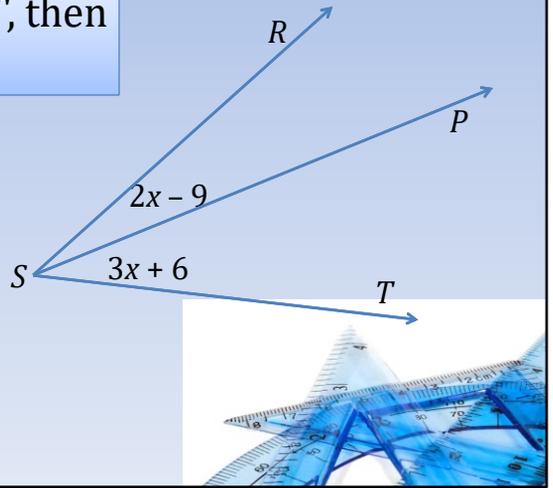
$\angle PQR$, $\angle PQS$, $\angle RQS$;
 $\angle PQS$ is a right angle .

1.5 Measuring and Constructing Angles

Angle Addition Postulate

If P is in the interior of $\angle RST$, then
 $m\angle RST = m\angle RSP + m\angle PST$

- If $m\angle RST = 72^\circ$, find $m\angle RSP$ and $m\angle PST$



$$2x - 9 + 3x + 6 = 72$$

$$5x - 3 = 72$$

$$5x = 75$$

$$x = 15$$

$$m\angle RSP = 2(15) - 9 = 21^\circ$$

$$m\angle PST = 3(15) + 6 = 51^\circ$$

1.5 Measuring and Constructing Angles

What is it?

Comparing shapes

Congruent Angles

$$\angle ABC \cong \angle DEF$$

What are examples?



What is it like?

Angles with same measure

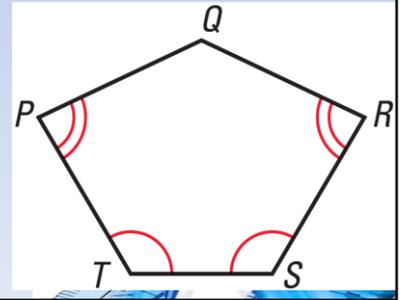
\cong means angles are exactly alike

= means measures are equal



1.5 Measuring and Constructing Angles

- Identify all pairs of congruent angles in the diagram.
- In the diagram, $m\angle PQR = 130^\circ$, $m\angle QRS = 84^\circ$, and $m\angle TSR = 121^\circ$. Find the other angle measures in the diagram.



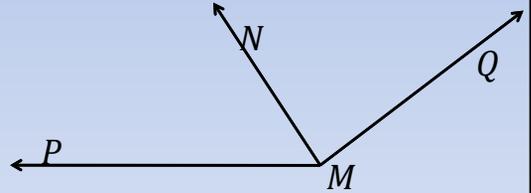
$$\angle T \cong \angle S, \angle P \cong \angle R$$

$$m\angle PTS = 121^\circ, m\angle QPT = 84^\circ$$

1.5 Measuring and Constructing Angles

Angle Bisector is a ray that divides an angle into two angles that are congruent.

- \overrightarrow{MN} bisects $\angle PMQ$, and $m\angle PMQ = 122^\circ$. Find $m\angle PMN$.



- 41 #2, 4, 6, 8, 10, 12, 16, 18, 20, 22, 24, 26, 28, 32, 34, 36, 38, 46, 61, 69



Objectives: By the end of the lesson,

- I can identify complementary and supplementary angles.
- I can identify linear pairs and vertical angles.
- I can find angle measures in pairs of angles.

1.6 DESCRIBING PAIRS OF ANGLES



1.6 Describing Pairs of Angles

What is it?

Angles Pairs

Adjacent Angles

What is it like?

Angles that share a ray and vertex

Are next to each other

Are not inside each other



What are examples?



1.6 Describing Pairs of Angles

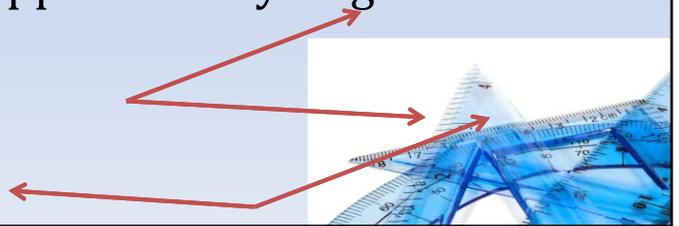
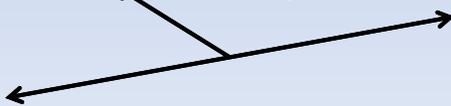
Complementary Angles

Two angles whose sum is 90°

Supplementary Angles

Two angles whose sum is 180°

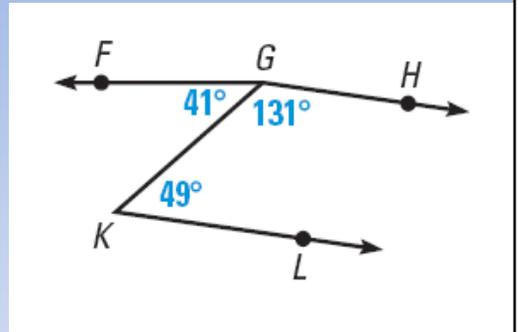
- Complementary and Supplementary Angles do not have to be adjacent



Both the pairs are supplementary

1.6 Describing Pairs of Angles

- In the figure, name a pair of
 - complementary angles,
 - supplementary angles,
 - adjacent angles.



- Are $\angle KGH$ and $\angle LKG$ adjacent angles?
- Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.



Complementary: $\angle FGK$ and $\angle GKL$

Supplementary: $\angle HGK$ and $\angle GKL$

Adjacent: $\angle FGK$ and $\angle HGK$

No, they do not have a common vertex

No, they are inside of each other

1.6 Describing Pairs of Angles

- Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 8^\circ$, find $m\angle 1$.
- Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 117^\circ$, find $m\angle 4$.



$$8 + x = 90 \rightarrow x = 82$$

$$117 + y = 180 \rightarrow y = 63$$

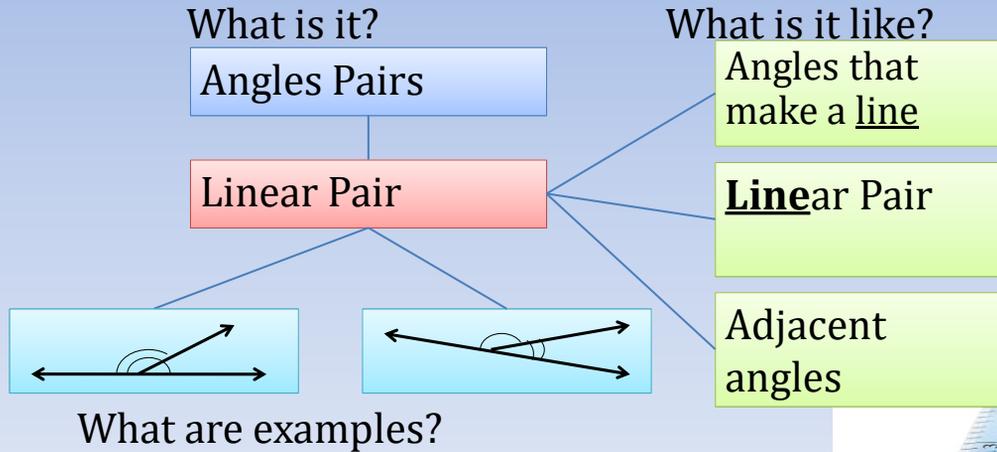
1.6 Describing Pairs of Angles

- $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles if $m\angle LMN = (4x - 2)^\circ$ and $m\angle PQR = (9x + 1)^\circ$.



$$\begin{aligned}(4x - 2) + (9x + 1) &= 90 \rightarrow 13x - 1 = 90 \rightarrow 13x = 91 \rightarrow x = 7 \\ m\angle LMN &= 4(7) - 2 = 26^\circ \\ m\angle PQR &= 9(7) + 1 = 64^\circ\end{aligned}$$

1.6 Describing Pairs of Angles



1.6 Describing Pairs of Angles

What is it?

Angles Pairs

Vertical Angles

What is it like?

Angles formed when 2 lines cross

On opposite sides of the intersection

Are not necessarily above each other

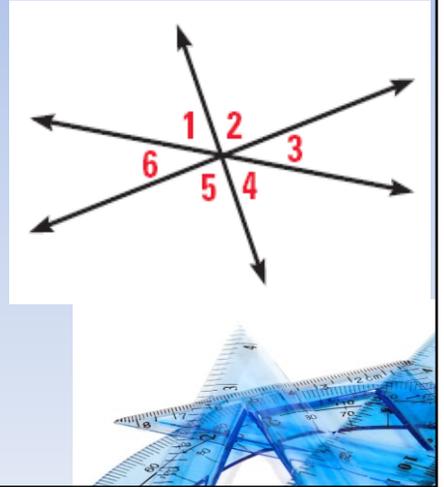


What are examples?

Vertical Angles are congruent.

1.6 Describing Pairs of Angles

- Do any of the numbered angles in the diagram below form a linear pair?
- Which angles are vertical angles?



No, no 2 of them form straight lines

$\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 6$

1.6 Describing Pairs of Angles

- Two angles form a linear pair. The measure of one angle is 3 times the measure of the other. Find the measure of each angle.



2

$$x + 3x = 180 \quad 4x = 180 \rightarrow x = 45 \rightarrow \text{angles are } 45 \text{ and } 135$$

1.6 Describing Pairs of Angles

- Things you can assume in diagrams.
 - Points are coplanar
 - Intersections
 - Lines are straight
 - Betweenness
- Things you cannot assume in diagrams
 - Congruence unless stated
 - Right angles unless stated
- 50 #2, 4, 6, 8, 10, 12, 14, 16, 20, 22, 24, 26, 28, 40, 42, 51, 52, 53, 54, 62

